

Please find attached, a sample test (and answers) given to us by Diablo Valley College which is being provided to you as an aid in preparation for your upcoming exam. It has been suggested by DVC that it is also a reflection of what you might find on the test itself.

Thank you and Good Luck!

Please note that this is not endorsed by UA Local 342's Apprenticeship Program, as we do not see the actual exam itself, and therefore cannot verify the similarities or accurateness.

Numerical Ability Review and Preparation

I. Operations with decimals

Addition and subtraction rules

Always line up the decimal point when adding decimals.

Example 1. Find the sum $4.2 + 9.25 + 18$

4.20
9.25
+18.00
31.45

Example 2. A residential home standing pressure is set at 79.82 psi but measures 84.9 psi. How much higher is the reading from the set value?

84.90
-79.82
5.08

Practice 1. John goes on four deliveries Monday morning and records 6.01 miles, 9.2 miles, 8 miles and, 24.63 miles. Find the total miles for these four deliveries.

Practice 2. The length of a $\frac{3}{4}$ inch copper pipe is 38.25 feet long but should be 37.29 feet long. How much shorter is the copper pipe?

Practice 3. Arrange these three amounts in descending order: .012, .12, and 1.2.

Multiplication rules $a \times b$, the answer is called the product

Step 1. Multiply the two numbers just as if they were whole numbers

Step 2. No need to align the decimal point. Place the decimal point in the answer by moving decimal point equal to the sum of the decimal points of both numbers.

Example 3. Find the product 37.7 and 2.81

37.7
× 2.81
377
3016
754
105.937

Example 4. Find the product of 10.25 and .0013

10.25
x.0013
3075
1025
.013325

Practice 4. Multiply 11.2 and 9.06

Practice 5. Find 1.1^8

Division rules: $a \div b$, b is the divisor and a is the dividend, the answer is called the quotient

Step 1. Place the divisor on the left of long division, and the dividend inside.

Step 2. Move the decimal point of the divisor to right obtain a whole number.

Step 3. Move the decimal point of the dividend the same number of places.

Step 4. Enter the decimal point in the quotient from step 3.

Step 4. Perform long division and continuing dividing until quotient repeats or ends.

Example 5. Find the quotient of $.2236 \div .043$

$.043 \overline{) .2236} \Rightarrow 43 \overline{) 223.6}$
5.2
$43 \overline{) 223.6}$
<u>215</u>
86
<u>86</u>
0

Example 6. Find the quotient $.02 \div 3$

$.003 \overline{) .02} \Rightarrow 3 \overline{) 20.}$
↓
6.6666...
$3 \overline{) 20.000}$
<u>18</u>
20

Practice6. $.0315 \div .0025$

Practice 7. A bag of quarters weighs 8.19 troy ounces. If a quarter weighs .182 troy ounces, how many quarters are there in the bag?

Verbal problems involving decimals

Example 7. Jose must work an average of 4.2 hours a day over the next three days. If Jose worked 3.5 hours on Monday, 2.1 hours on Tuesday, how many hours does he need to work on Wednesday?

Solution. If the average is 4.2 hours a day, then Jose must work a total of $3 \times 4.2 = 12.6$ hours for the next three days. Monday and Tuesday totals 5.6, so Jose needs to work 7 hours on Wednesday.

Example 8. You need to put some gasoline in your truck. Gasoline is \$4.85 a gallon. You only have a total of \$50.44 in your pocket. How many gallons can you buy?

Divide \$50.44 by 4.85

$$\begin{array}{r} 10.4 \\ 485 \overline{) 5044.00} \\ \underline{485} \\ 1940 \\ \underline{1940} \\ 0 \end{array}$$

Example 9. Jackie stops to buy some a 6pack of energy drinks costing \$9.40 plus a deposit of \$.60, a bag of chips for \$2.50, and 4 bananas costing .23 each. How much will she receive in change from a \$20 bill?

Drink	9.40
deposit	.60
chips	2.50
4 bananas	.92
	\$13.42
Jackie should get back $\$20.00 - \$13.42 = \$6.58$	

Practice 8. A truck hauling 82 crates of steel roads in which each crate weighs 50.4 pound is stopped for excessive weight as the truck has a limit of 4,000 pounds. How many crates must be removed to be under the limit.

Practice 9. A floor is 15.25 feet wide. Square tiles 8 inches square are to span across the floor. How many whole tiles are needed and how wide in inches is the remaining space that needs to be filled?

Practice 10. Harry bought two heads of cabbage for \$1.80. How many heads of cabbage can Harry buy if he has \$28.80?

Solutions to practice problems involving operations with decimals

- Practice 1 47.84
- Practice 2 .96
- Practice 3. 1.2, .12, .012
- Practice 4. 101.472
- Practice 5. 1.331
- Practice 6. 12.6
- Practice 7. 45
- Practice 8. 3 crates
- Practice 9. 22 tiles are needed with a space of 7 inches of space needs to be filled.
- Practice 10. 32

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II. Operations with rational expressions

A. Addition and subtraction of fractions

To add fractions, be sure that the denominators are the same. When the denominators are different, build each fraction so the denominators of each fraction are the least common multiple.

Example 1. Combine $\frac{3}{16} + \frac{2}{3} - \frac{1}{24}$

Combine $\frac{3}{16} + \frac{2}{3} - \frac{1}{24}$, the lcm is 48
$\frac{9}{48} + \frac{32}{48} - \frac{2}{48} = \frac{39}{48} = \frac{13}{16}$

Example 2. Tom decides to paint the family room. On Wednesday, he paints $\frac{2}{15}$ of the room, on Thursday, he paints $\frac{1}{8}$ of the room, and on Friday, he paints $\frac{1}{5}$ of the room. If he finished painting the room on Saturday, how much does he need to paint?

Tom needs to paint $1 - \frac{2}{15} - \frac{1}{8} - \frac{1}{5}$, the lcm is 120
$\frac{120}{120} - \frac{16}{120} - \frac{15}{120} - \frac{24}{120} = \frac{65}{120} = \frac{13}{24}$

Practice 1. Alice runs a marathon Sunday morning. She runs $\frac{3}{10}$ of the marathon the first hour, $\frac{1}{4}$ of the marathon the 2nd hour, and $\frac{2}{5}$ of the marathon the 3rd hour. How much does she have left to run?

Practice 2. Felix pumps gas into his car every morning. On Monday he pumps $8\frac{3}{8}$ gallons, on Tuesday he pumps $10\frac{1}{5}$ gallons, and on Wednesday he pumps $4\frac{1}{2}$ gallons. If gas cost \$4.00 a gallon, how much did Felix spend on gas?

B. Converting between decimals to fractions

Method 1. To convert fractions to decimals divide denominator into the numerator.

Method 2. To convert decimals to fractions, expression a fraction using place value.

Example 3. Write 32.125 as a fraction

.125 has a place value of thousandth.
$32.125 = 32 + \frac{125}{1000} = 32 + \frac{1}{8} = 32\frac{1}{8}$

Example 4. Express $3\frac{1}{7}$ as a decimal

$3\frac{1}{7} = \frac{22}{7} \Rightarrow 7 \overline{)22.000}$	$3.14\frac{2}{7}$
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Example 5. Express $4\frac{5}{16}$ as a terminating decimal

$4\frac{5}{16} = \frac{69}{16}$
4.3125
$16 \overline{)69.000}$
<u>64</u>
50
<u>48</u>
20
<u>16</u>
40
<u>32</u>
80
<u>80</u>

Example 6. Express 2.3071 as a fraction

2.3071 represent $2 + \frac{3071}{10,000} = \frac{23071}{10000}$
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Practice 3. Which is larger 3.14 or $\frac{22}{7}$?

Practice 4. Write sum as a fraction $2.10 + \frac{1}{8} + \frac{1}{4}$

C. Verbal problems involving fractions

1. Ratio is expressed as a fraction $\frac{a}{b}$ or a:b
2. A proportion is an equation between two ratios: $\frac{a}{b} = \frac{c}{d}$
3. Given $\frac{a}{b} = \frac{c}{d}$, the product of the means equal the product of the extremes $b c = a d$
4. Solving first degree equations if $ax=b$ then $x = \frac{b}{a}$

Example 7. There are 520 applications for a job opening in which there are 340 males. Find the ratio of females to males.

There are 340 males and 180 females.
The ratio of females to males is $\frac{180}{340}$ or $\frac{9}{17}$

Example 8. A mixture of grout requires for 20 ounces of cement for every 5 gallons of water. Use ratios and proportions to find how many ounces of cement is required for 18 gallons of water? Express answer as a mixed fraction.

The ratio of cement to water is $\frac{20 \text{ ounces}}{5 \text{ gallons}}$.

$$\frac{20}{5} = \frac{c}{18}$$
$$20 \cdot 18 = 5c$$
$$360 = 5c$$
$$c = 72 \frac{3}{5} \text{ ounces of cement.}$$

Example 9. Use the fact that 60 miles per hour is exactly 88 feet per second to convert 100 miles per hour to feet per second.

$\frac{60 \text{ m/h}}{88 \text{ ft/sec}} = \frac{100 \text{ m/h}}{x \text{ ft/sec}}$ $60x = 8800$ $x = \frac{8800}{60} = 146.67 \text{ ft/sec}$
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Practice 6. For every 40 miles of freeway, there are 9 potholes on the road. find how many potholes there are in 300 miles of freeway. Express answer as a decimal to the nearest hundredth.

Practice 7. Determine the distance between city A and City B using a map. The map key indicates that 5 cm is equivalent to 24 km. If the cities are 18 km apart on the map, what is the distance on the map between the cities expressed as a mixed fraction?

Practice 8. Convert 120 miles per hour to feet per second.

Practice 9. Jerry used his computer to reduce the size of a picture to a width of $3\frac{3}{10}$ in. What is the new height if it was originally $32\frac{1}{2}$ in tall and $42\frac{9}{10}$ in wide?

Solutions to practice problems involving rational expressions

- Practice 1 $\frac{1}{20}$
- Practice 2 \$92.30
- Practice 3. $\frac{22}{7}$
- Practice 4. $2\frac{19}{40}$
- Practice 5. 4.3125
- Practice 6. 675
- Practice 7. $3\frac{3}{4} \text{ cm}$
- Practice 8. 176
- Practice 9. $2\frac{1}{2}$

III. Operations with percent

A. Basic Conversions

- Skill 1. To convert decimal to a percent, move decimal point two places to the right and affix %
- Skill 2. To convert a fraction to a percent, divide denominator into numerator and execute skill 1.
- Skill 3. To convert a percent written in fraction form, drop the % sign and multiply by $\frac{1}{100}$.
- Skill 4. To convert a percent written in decimal form, drop the % sign and multiply by .01

Example 1. Convert 2.3% to a decimal

$$2.3\% = 2.3 \cdot .01 = .023$$

Example 2. Convert $15\frac{2}{3}\%$ to a fraction

$$15\frac{2}{3}\% = 15\frac{2}{3} \cdot \frac{1}{100} = \frac{47}{3} \cdot \frac{1}{100} = \frac{47}{300}$$

Example 3. Convert $\frac{3}{400}$ to a percent

$$\frac{3}{400} = .0075 = .75\% \text{ or}$$
$$\frac{3}{400} = \frac{3}{4} \cdot \frac{1}{100} = \frac{3}{4}\%$$

Example 4. Convert .153 to a percent

$$.153 = 15.3\%$$

Practice 1. Convert $\frac{2}{3}$ to a percent.

Practice 2. Convert the sum to a percent, $2.3\% + \frac{3}{4} + .01$

Practice 3. Convert $14\frac{1}{8}\%$ to a fraction

Practice 4. convert 8.24% to a decimal.

B. Verbal Percent problems

- **Skill 1. To determine the percent increase, determine the base price, the increase price and divide the base price into the increased price.**
- **Skill 2. To determine what value is ___% of a given value, change the ___% to a fraction or decimal and multiply fraction or decimal to the given value.**
- **Skill 3. To determine the ___% of an unknown value is a given number, divide the given number by the ___%.**

Example 5. Clyde has a 40 hour workweek, but next week is scheduled to work $8\frac{2}{3}\%$ more hours. How many hours is Clyde scheduled to work? Express answer rounded to the nearest cents.

$$\begin{aligned} &\text{Clyde is scheduled to work } 108\frac{2}{3}\% \text{ hours next week} \\ &= 108\frac{2}{3}\% \text{ of } 40 \text{ hours} \\ &= \frac{326}{3}\% \text{ of } 40 \text{ hours} \\ &= \frac{326}{300} \cdot 40 = \frac{163}{150} \cdot 40 = \frac{163}{15} \cdot 4 \\ &= 43.47 \text{ hrs} \end{aligned}$$

<p>Let x = Janis old salary</p> <p>Janis new salary is 130% of her old salary.</p> <p>$1.30x = \\$42,000$</p> <p>$x = \frac{42000}{1.3} = \\$32,307.69$</p>
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Example 6. Janis worked full time as an apprentice and earned a college degree at night. Her company rewarded Janis with a new salary of \$42,000 a year which is a 30% increase based on her old salary. What was Janis old salary rounded to the nearest cent

Example 7. Big Rock Metal made the lowest bid of \$27,200 on a fabrication job. This bid was 15% lower than then the second lowest bid. How much was the second lowest bid?

<p>Let x = 2nd lowest bid</p> <p>$.85x = \\$27,200$</p> <p>$x = \frac{27200}{.85} = \\$32,000$</p>

Practice 5. Purchasing a new garbage disposal costing \$245, the sales tax is 8% of the purchase price. How much was the sales tax?

Practice 6. The Solar Bright Company decided to increase the workforce by 18% to a total workforce of 3,776 workers. What was the workforce before the increase?

Practice 7. A refrigerator store uses a 40% markup on cost. Find the cost of a refrigerator that sells for \$2,940.

Practice 8. 420 students in a class took an apprentice test. If 1336 students passed the test, what percent do not pass?

Solutions to practice problems involving percent

- Practice 1 $66\frac{2}{3}\%$

- Practice 2 2.3%+76%=78.3%
- Practice 3. $\frac{113}{800}$
- Practice 4. .0824
- Practice 5. \$19.60
- Practice 6. 3,200
- Practice 7. \$2,100
- Practice 8. 20%

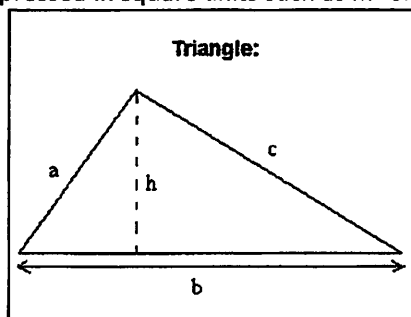
IV. Perimeters and Areas of Plane figures

Let P denote the perimeter and A denote the area. Perimeters expressed in units such as centimeters, meters, inches or feet and areas are expressed in square units such as m² or ft².

A. Triangles- $P=a+b+c$ and $A=\frac{1}{2}b \cdot h$

Example 1. Find the perimeter of a triangle with sides 3.4 cm, 4.2 cm and 5.1 cm.

$$\begin{aligned} \text{Perimeter} &= 3.4 + 4.2 + 5.1 \\ &= 12.7 \text{ cm} \end{aligned}$$



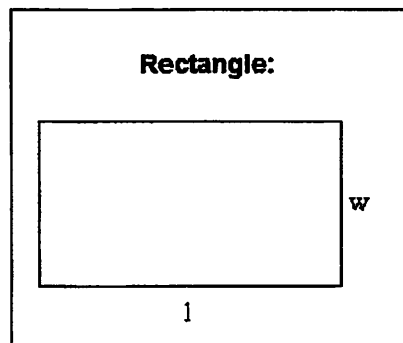
Example 2. Find the area of a triangle whose base is 10 m and height(altitude) is $3\frac{3}{4}$ m.

$$\begin{aligned} \text{Area} &= \frac{1}{2}(10)\left(3\frac{3}{4}\right) \text{ sq meters} \\ &= \frac{1}{2}(10)\left(\frac{15}{4}\right) \text{ m}^2 = \frac{75}{4} \text{ m}^2 = 18.75 \text{ m}^2 \end{aligned}$$

B. Rectangle: $P=2l+2w$ and $A=lw$

Example 3. Find the perimeter of a rectangle whose length is $3\frac{2}{3}$ feet and whose width is $2\frac{1}{2}$ feet.

$$\begin{aligned} P &= 2l + 2w \\ &= 2 \cdot 3\frac{2}{3} + 2 \cdot 2\frac{1}{2} \\ &= 2 \cdot \frac{11}{3} + 2 \cdot \frac{5}{2} = \frac{22}{3} + 5 \end{aligned}$$

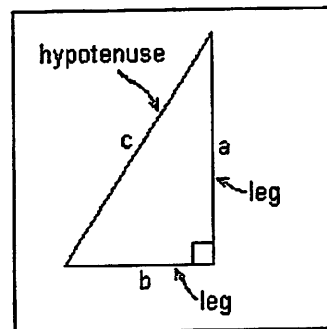


Example 4. Find the area of a rectangle whose length is $3\frac{2}{3}$ feet and whose width is $2\frac{1}{2}$ feet.

$$\begin{aligned} \text{Area } A &= lw \\ &= 3\frac{2}{3} \cdot 2\frac{1}{2} \\ &= \frac{11}{3} \cdot \frac{5}{2} = \frac{55}{6} \\ &= 9\frac{1}{6} \text{ square feet} = 9\frac{1}{6} \text{ ft}^2 \end{aligned}$$

C. Pythagorean Theorem: in a right triangle whose legs are a and b , and hypotenuse is c then $a^2 + b^2 = c^2$

Example 5. Find the hypotenuse of a right triangle whose legs measure 12 inches and 5 inches.



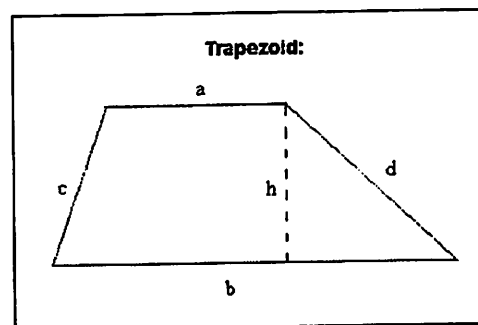
$$\begin{aligned} a &= 12 \text{ in}, b = 5 \text{ in}, \text{ find side } c. \\ c^2 &= a^2 + b^2 \\ c^2 &= 12^2 + 5^2 \\ c^2 &= 144 + 25 = 169 \\ c &= 13 \text{ inches} \end{aligned}$$

D. Trapezoid is a four-sided polygon in which a pair of opposite sides is parallel.

$$\text{Perimeter} = a + b + c + d$$

$$\text{Area} = \frac{1}{2}(a+b)h$$

Example 6. Find the area of a trapezoid in which the pair of parallel sides are $7\frac{1}{2}$ feet and 8.3 feet, and whose height is 10 feet.



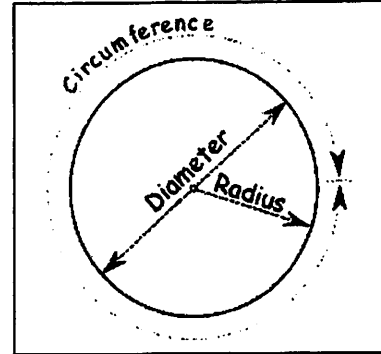
$$\begin{aligned} A &= \frac{1}{2}(7\frac{1}{2} + 8.3) \cdot 10 \text{ square feet} \\ &= .5(7.5 + 8.3) \cdot 10 \\ &= .5(15.8)(10) = 79 \text{ ft}^2 \end{aligned}$$

E. Circles

Perimeter= circumference = $\pi d = 2\pi r$, where d =diameter and r =radius

Area = πr^2

Use for approximate of $\pi \approx 3.14$ or $\frac{22}{7}$



Example 7. Find the circumference of a circle whose radius is 3.1 meters. Use $\pi \approx 3.14$

$$\begin{aligned} C &= 2\pi r \\ &= 2(3.14)(3.1) = 19.468 \text{ m} \end{aligned}$$

Example 8. Find the area of a circle whose diameter is $4\frac{2}{3}$ meters, Use $\pi \approx \frac{22}{7}$

$$\begin{aligned} A &= \pi r^2 \\ &= \frac{22}{7} \left(4\frac{2}{3}\right)^2 \\ &= \frac{22}{7} \left(\frac{14}{3}\right)^2 \\ &= \frac{22}{7} \left(\frac{14}{3}\right) \left(\frac{14}{3}\right) = \frac{616}{9} \\ &= 68\frac{4}{9} \text{ m}^2 \end{aligned}$$

Practice 1. Find the perimeter of a square whose side is $3\frac{4}{5}$ meters.

Practice 2. Find the area of a triangle whose base is 12 feet and height is 10 feet.

Practice 3. Find the circumference of a circle whose diameter is 10 inches, use $\pi \approx 3.14$

Practice 4. Find the area of a circle whose diameter is 10 inches, use $\pi \approx \frac{22}{7}$

Practice 5. Find the hypotenuse of a right triangle whose legs are 30 cm and 40 cm.

Practice 6. Find the area of a rectangle whose length is $3\frac{4}{5} f$ and whose width is $2\frac{3}{4} f$

Practice 7. Find the perimeter of a rectangle whose length is $3\frac{4}{5}f$ and whose width is $2\frac{3}{4}f$.

Practice 8. Compute the area of a trapezoid whose parallel sides measure 15 inches and 13 inches. the altitude measures 5 inches.

Solutions to practice problems involving perimeters and areas

- Practice 1 $15\frac{1}{5}m$
- Practice 2 60 ft^2
- Practice 3. 31.4 in
- Practice 4. $78\frac{4}{7}\text{ in}^2$
- Practice 5. 50 cm
- Practice 6. 10.45 ft
- Practice 7. 13.1 ft
- Practice 8. 70 in^2

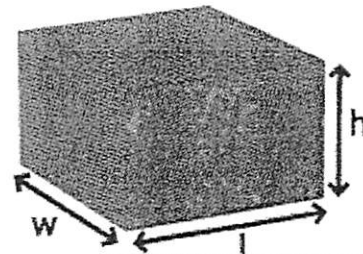
V. Volume of Solids

Units are expressed in cubic units such as m^3 , and ft^3 .

- A. Rectangular Prism
Given w =width, l =length,
and h =height

$$\text{Area} = 2lw + 2lh + 2wh$$

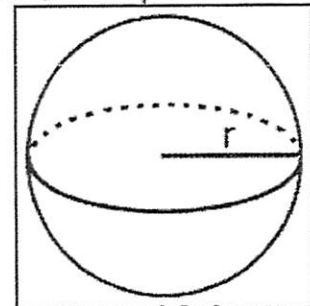
$$\text{Volume} = lwh$$



- B. Sphere
Given r =radius of sphere,

$$\text{Area} = 4\pi r^2$$

$$\text{Volume} = \frac{4}{3}\pi r^3$$

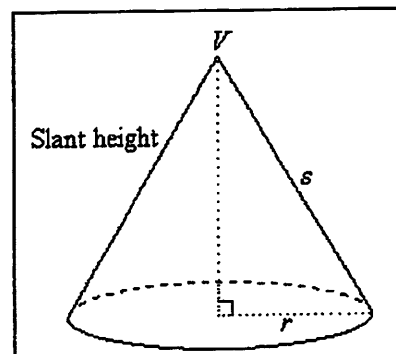


C. Cone

Given r =radius and h =height

$$Area = \pi r(r + s)$$

$$Volume = \frac{1}{3}\pi r^2 h$$



Example 1. Find the volume and surface area of sphere whose diameter is 3 inches, use $\frac{22}{7}$ as approximation for π .

$$\text{Given } r = 1\frac{1}{2} = \frac{3}{2}$$

$$V = \frac{4}{3}\pi r^3 \Rightarrow V = \frac{4}{3} \cdot \frac{22}{7} \left(\frac{3}{2}\right)^3 = \frac{4}{3} \cdot \frac{22}{7} \left(\frac{27}{8}\right) = \frac{99}{7} = 14\frac{1}{7} \text{ cubic inches}$$

$$A = 4\pi r^2 \Rightarrow A = 4 \cdot \frac{22}{7} \left(\frac{3}{2}\right)^2 = 4 \cdot \frac{22}{7} \left(\frac{9}{4}\right) = \frac{198}{7} = 28\frac{2}{7} \text{ sq inches}$$

Example 2. How much water is in a PVC pipe whose diameter is 1 foot and whose length is 20 feet. Assume that 7.48 U.S. gallons in a cubic foot and round to the nearest tenths using 3.14 for the approximation to π .

The PVC pipe is a cylinder with $d=1$ ft, $r=.5$, and $h=20$ ft

$$\text{Volume} = \pi r^2 h = 3.14(.5^2)(20) \text{ ft}^3 = 15.7 \text{ ft}^3$$

$$\text{Number of gallons} = \text{volume} \cdot (\text{gallons per cubic foot}) = 15.7 \cdot 7.48 = 117.4 \text{ gallons}$$

Example 3. The top of a water heater is in the shape of a cone whose diameter is 14 inches, and height 24 inches. Find the area of the cone including the base, and find the total volume.

<p>The radius, r, of the cone is 7 inches, and the $h=24$ inches</p> <p>Using the Pythagorean Theorem, $s^2 = r^2 + h^2$</p> $s^2 = 7^2 + 24^2 = 625$ $s = 25 \text{ inches}$ $\text{Area} = \pi r(r+s) = \frac{22}{7} \cdot 7(7+25) = 682 \text{ in}^2$ $\text{Volume} = \frac{1}{3} \pi r^2 h = \frac{1}{3} \cdot \frac{22}{7} (7^2)(24) \text{ in}^3 = 1,232 \text{ in}^3$
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Example 4. Two size rectangular boxes are sold at a store: The Skinny One measures 6 inches by 4 inches by 20 inches. The Fat One measures 6 inches by 8 inches by 10 inches. Compute the volume and decide which box uses the least cardboard.

<p>The volume of Skinny box using $V = lwh = 6(4)(20) \text{ in}^3 = 480 \text{ in}^3$</p> <p>The volume of Fat box using $V = lwh = 6(8)(10) \text{ in}^3 = 480 \text{ in}^3$.</p> <p>The surface area $= 2lh + 2lw + 2wh$</p> <p>Skinny Box $A = 2(6)(20) + 2(6)(4) + 2(4)(20) = 448 \text{ in}^2$.</p> <p>Fat Box $A = 2(6)(10) + 2(6)(8) + 2(8)(10) = 376 \text{ in}^2$.</p> <p>Fat Box more environmentally conscious.</p>
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VI. Patterns and Sequences

To determine a pattern for a sequence involves a series of systematic guessing and testing. Look for addition, multiplication and raising previous number by powers.

Example 1. What is the next three numbers in this sequence
 1, 2, 4, 7, 11, __, __, __

Notice that the difference between consecutive numbers are 1,2,3,4.
So you would expect the pattern to continue. 16,22,29

Example 2. What is the next three numbers in this sequence
3,9,27,81,, _ _ _

Notice that the next number in the sequence is found by multiplying by 3.
So you would expect the pattern to continue. 243,729 2187

Example 3. What is the next three numbers in this sequence
2,5, 8, 11,, _ _ _

Notice that the next number in the sequence is found by adding 3.
So you would expect the pattern to continue. 14, 17, 20

Example 4. What is the next three numbers in this sequence
1, 1, 2, 3, 5, 8,, _ _ _

Notice that the next number in the sequence is found by
adding the previous two numbers.
So you would expect the pattern to continue.
13, 21, 34

Example 5. What is the next three numbers in this sequence
1, 4, 9, 16, 25,, _ _ _

Notice that the next number in the sequence is found by
squaring the location of the number in the sequence.
So you would expect the pattern to continue.
 $6^2, 7^2, 8^2$ or 36, 49,64

VII. Logical Thinking

Be creative when solving logical puzzles. Practice with puzzles found in magazines and newspapers. Be patient, and let the problem incubate and soon a sudden solution will appear. Always check your answer to be sure you are correct.

Example 1. If you have a penny, quarter, and a half dollar, how many different amounts can you form using at least one coin.

To solve this problem systematically list the different amounts you can form with one coin, two coins, and all three coins.

1 coin: 1,25,50

2 coins: 26,51,75

3 coins;76

So the answer is 7 amounts.

Example 2. At a party, each person shook with every other person exactly once. If there were 6 handshakes, how many people were at the party.

Determine how many handshakes if there were only 2 people, only 3 people, only 4 people, and so on.
Did you get answer of 6 people?

Example 3. A custodian has four keys to open four locked offices. Unfortunately, the custodian did not label the keys and are mixed up. What is the maximum number of attempts the custodian must make before all four offices are opened.

Again, logically attempt to open the first door.
What is the maximum number of attempts that can be made?
Did you say 4? Attempting to open the 2nd door, remember door 1 is opened.
How many attempts maximum are required for door 2? Did you say 3?
Continuing, you would determine $4+3+2+1=10$

Example 4. Find the sum of all the divisors of 100.

Systematically list all possible divisors: 1, 2, 4, 5, 10, 20, 25, 50, 100.
Next, sum these values 217

Example 5. A plane leave the North Pole and flies due south for 100 miles. Then the plane turns and flies west for 200 miles. At the end of that trip, how far is the plane from the North Pole.

Visually imagine a plane leaving from the North Pole flying the indicated directions. Did you get an answer of 100 miles?

VIII. Work and Rates

Solving problems involving work uses two basic measures: The rate which represent how much of the work has been finished divided by some unit of time. Examples such as the rate a painter painting a house can be measured as $\frac{1}{5}$ of the house painted per day, the rate a faucet filling a sink can be $\frac{2}{3}$ of sink filled in 10 minutes, or a installer lays 100 feet of pipe hour.

Basic Formula 1: If it takes n units of time to finish one job, then the rate is $\frac{1}{n}$ of job finished in one unit of time.

Basic Formula 1: If t =time worked on a job and the rate is $\frac{1}{n}$ then the work done is $t \cdot \frac{1}{n}$

Example 1. The cold water faucet can fill the bath tub in 12 minutes. The hot water faucet can fill the same bath tub in 15 minutes. Find how long it would take to fill the bath tub if both faucets are turned on and function independently.

The rate of the cold water faucet is $\frac{1}{12}$ of basin in a minute.

The rate of the hot water faucet is $\frac{1}{15}$ of basin in a minute

let t = time both faucets are turned on

$$\frac{1}{12}t + \frac{1}{15}t = 1$$

$$\frac{5}{60}t + \frac{4}{60}t = 1$$

$$\frac{9}{60}t = 1$$

$$t = \frac{60}{9} = \frac{20}{3} = 6\frac{2}{3} \text{ min}$$

Example 2. Marlo and Ellen must install venting in an old house. Marlo can do the job alone in 4 days. Ellen can do the entire job in 3 days. They both start out working together, but Ellen quits after working one day having found a better job. How many days did Marlo work from the start to finish the entire job.

The rate for Marlo is $\frac{1}{4}$ of job in a day.

The rate for Ellen is $\frac{1}{3}$ of job in a day.

let t = time Marlo worked on job

$$\frac{1}{4}t + \frac{1}{3}(1) = 1$$

$$\frac{1}{4}t = \frac{2}{3}$$

$$\frac{9}{60}t = 1$$

$$t = \frac{8}{3} = 2\frac{2}{3} \text{ days}$$